

Single neutron hole entropy in  $^{105}\text{Cd}$  and  $^{111}\text{Cd}$ Z. Moradipoor<sup>1</sup> and R. Razavi<sup>2,\*</sup><sup>1</sup>Physics Group, North Tehran Branch, Islamic Azad University, Tehran, Iran<sup>2</sup>Department of Physics, Faculty of Science, Imam Hossein Comprehensive University, Tehran, Iran

(Received January 1, 2015; accepted in revised form April 11, 2015; published online October 20, 2015)

The nuclear level density and entropy were calculated for  $^{105}\text{Cd}$ ,  $^{106}\text{Cd}$ ,  $^{111}\text{Cd}$  and  $^{112}\text{Cd}$  based on the Back Shifted Fermi Gas (BSFG) model and the Constant Temperature (CT) model. Then, the entropies were extracted in the microcanonical ensemble according to recent experimental data on nuclear level density measured by the Oslo group for these nuclei and are compared with their corresponding macroscopic calculations. Entropies of the neutron hole were estimated from the entropy difference between the odd-mass and even-even nuclei. The results reveal that the CT model describes better the extracted microcanonical results.

Keywords: Nuclear level density, Entropy, Single neutron hole, Microcanonical ensemble

DOI: [10.13538/j.1001-8042/nst.26.050503](https://doi.org/10.13538/j.1001-8042/nst.26.050503)

## I. INTRODUCTION

The nuclear level density  $\rho(E)$ , defined as the number of levels per unit energy at certain excitation energy, plays a crucial role in studying nuclear structure and is the starting point to deduce the thermodynamic quantities [1–3]. The first theoretical attempt to explain nuclear level densities was done by Hans Bethe in 1936 [4]. In doing so, the nucleus was described as a gas of non-interacting fermions moving in equally spaced single-particle orbitals. The Fermi-gas model was later modified by introducing a shift in the excitation energy  $E$  due to residual interactions between the nucleons. The pairing correlations in the low excitation region play a major role and are described within the so called Back-Shifted Fermi Gas (BSFG) model. The Constant Temperature (CT) model, introduced by Gilbert and Cameron [5], is another phenomenological model based on experimental evidence that the discrete levels follow exponential law.

These models are still widely used to describe nuclear level densities. The thermodynamic identities are the starting point for extracting results that are useful in relating different experimental quantities. An important goal in nuclear physics is to deduce thermodynamic quantities such as entropy and temperature [6, 7]. The information entropy in the field of information theory is introduced for studying the disassembly of nuclei in the framework of the isospin dependent lattice gas model and molecular dynamical model [8].

In this work, the nuclear level densities for  $^{105}\text{Cd}$ ,  $^{106}\text{Cd}$ ,  $^{111}\text{Cd}$  and  $^{112}\text{Cd}$  within the two phenomenological models are extracted according to the new experimental data measured by the Oslo group [9]. The entropy is deduced as a function of excitation energy for each of the nuclei. The theoretical results of level densities are compared with the experimental data.

## II. STATISTICAL FORMULAS

The nuclear temperature  $T$  and the level density are related by the expression [10]

$$1/T = d[\ln \rho(E)]/dE. \quad (1)$$

The integration yields the constant temperature level density [5]

$$\rho(E) = e^{(E-E_0)/T}/T, \quad (2)$$

where  $E_0$  is the ground state back-shift energy.

The Bethe formula of the nuclear level density for the back-shifted Fermi gas model [11, 12] can be written as:

$$\rho(E) = \frac{\exp(\sqrt{a(E-E_1)})}{12\sqrt{2}\sigma^4 \sqrt{a(E-E_1)}^5}, \quad (3)$$

where  $a$  is the level density parameter,  $E_1$  is the ground state back-shift energy and  $E$  is the excitation energy. Gilbert and Cameron calculated the spin cut-off parameter for the Bethe formula with the reduced moment of inertia [5]

$$\sigma^2 = 0.0888A^{2/3}[a(E-E_1)]^{1/2}. \quad (4)$$

The spin cut-off parameter is related to an effective moment of inertia  $I_{\text{eff}}$  and to the nuclear temperature  $T$

$$\sigma^2 = I_{\text{eff}}T/\hbar^2. \quad (5)$$

The level density for a system of  $N$  neutrons and  $Z$  protons is related to the state density by the relationship

$$\rho(N, Z, E) = \omega(N, Z, E)(2\pi\sigma^2)^{-1/2}. \quad (6)$$

The state density, which is the inverse Laplace transform of the grand partition function, is given by

$$\omega(N, Z, E) = e^S \cdot (2\pi)^{-3/2} \cdot D^{-1/2}, \quad (7)$$

where  $D$  is a  $3 \times 3$  determinant of the second derivations of the grand partition function taken at the saddle point.

\* Corresponding author, [rrazavin@ihu.ac.ir](mailto:rrazavin@ihu.ac.ir)

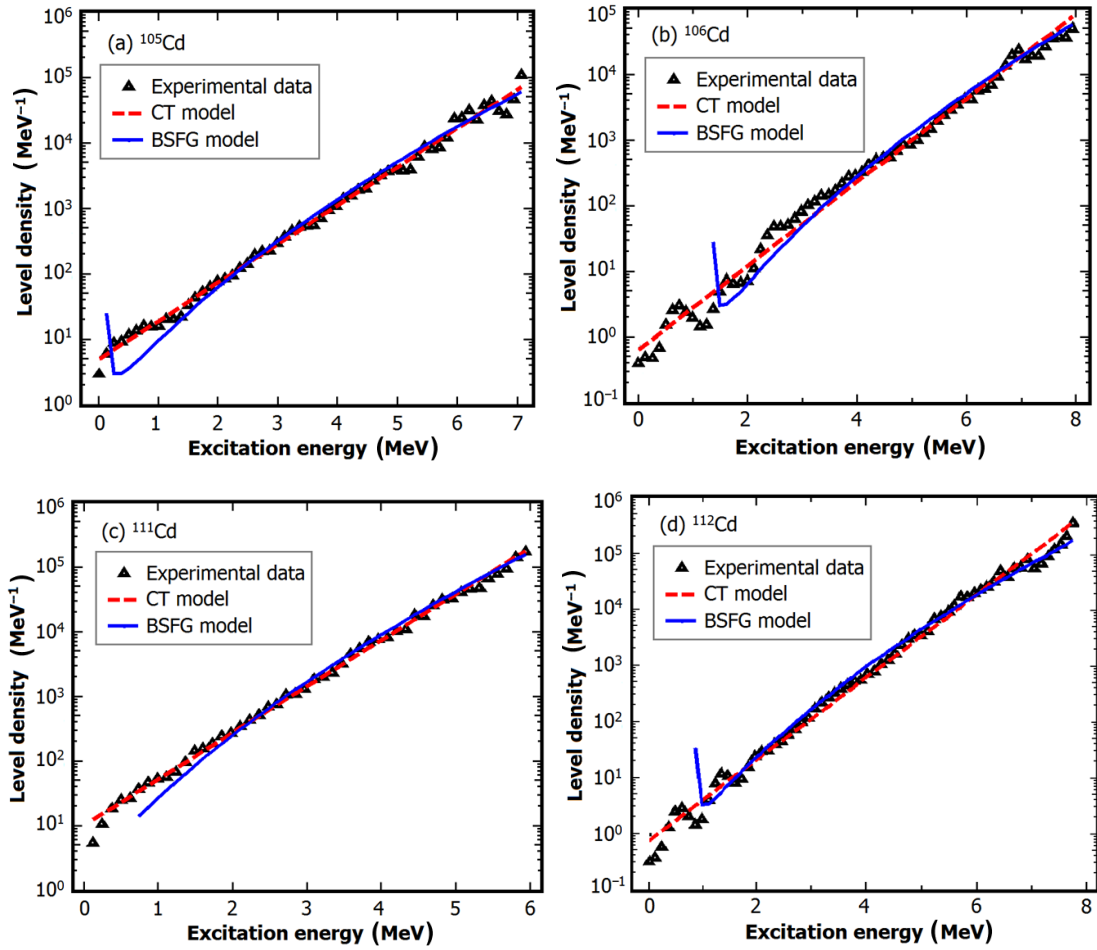


Fig. 1. (Color online) Experimental and calculated level density as a function of excitation energy in (a)  $^{105}\text{Cd}$ , (b)  $^{106}\text{Cd}$ , (c)  $^{111}\text{Cd}$  and (d)  $^{112}\text{Cd}$ .

The level density is proportional to the number of states accessible at a given excitation energy. The multiplicity of states defined as  $\rho/\rho_0$ , where  $\rho_0$  is the level density close to the ground state. Thus, the entropy in the microcanonical ensemble is given by

$$S = K_B \cdot \ln(\rho/\rho_0), \quad (8)$$

where  $K_B$  is the Boltzmann constant. The single neutron hole entropy is given by [13]

$$\Delta S = S(\text{Odd}A) - S(A+1). \quad (9)$$

### III. RESULTS AND DISCUSSION

Nuclear level densities of  $^{105}\text{Cd}$ ,  $^{106}\text{Cd}$ ,  $^{111}\text{Cd}$  and  $^{112}\text{Cd}$  are extracted in the CT and BSFG models, using the experimental data on nuclear level density measured by the Oslo group [9]. Two nuclear level density formulas are parameterized and the results are tabulated in Table 1. The results of level densities as a function of excitation energy are shown in Fig. 1. Their corresponding experimental values [9] are

TABLE 1. Level density parameters of BSFG and CT models for  $^{105}\text{Cd}$ ,  $^{106}\text{Cd}$ ,  $^{111}\text{Cd}$  and  $^{112}\text{Cd}$

Nuclei	BSFG		CT	
	$a(\text{MeV}^{-1})$	$E_1(\text{MeV})$	$T$	$E_0(\text{MeV})$
$^{105}\text{Cd}$	12.06	0.12	0.74	-0.98
$^{106}\text{Cd}$	12.65	1.36	0.68	-0.53
$^{111}\text{Cd}$	15.45	-0.01	0.61	-1.12
$^{112}\text{Cd}$	13.64	0.87	0.59	0.49

plotted for comparison. As can be seen from Fig. 1, the overall agreement between the experimental level densities and theoretical results is satisfactory. However, the CT model reproduces better level densities for these nuclei than the BSFG model.

According to Eq. (8), the entropy is calculated from the calculated level density. The entropies are also extracted from experimental data on nuclear level density in the microcanonical ensemble framework. The calculated entropies in the microcanonical ensemble are shown in Fig. 2 for  $^{111}\text{Cd}$  and  $^{112}\text{Cd}$ , respectively. Their corresponding values obtained from the phenomenological models are plotted for compar-

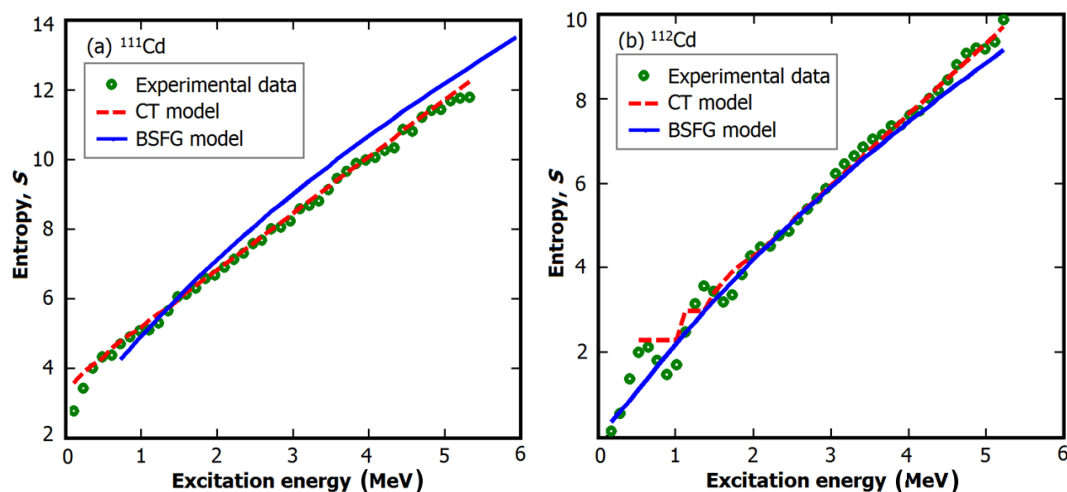


Fig. 2. (Color online) The entropy as a function of excitation energy for (a)  $^{111}\text{Cd}$  and (b)  $^{112}\text{Cd}$ .

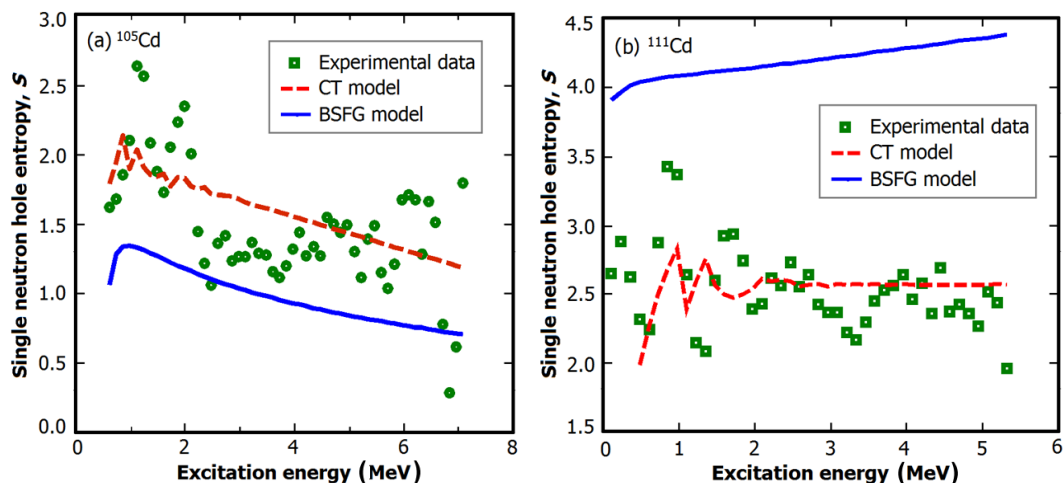


Fig. 3. (Color online) Calculated single neutron hole entropy in (a)  $^{105}\text{Cd}$  and (b)  $^{111}\text{Cd}$ .

ison. It can be seen that the microcanonical entropy corresponds well with the calculated entropy within the CT model. The entropy is related to the number of microstates accessible to a system. The entropy of  $^{105}\text{Cd}$  ( $^{111}\text{Cd}$ ) nucleus follows closely the entropy for  $^{106}\text{Cd}$  ( $^{112}\text{Cd}$ ), but the even-odd system has an entropy excess. The single neutron hole entropy is defined as the difference between the entropy of the even-odd and even-even neighbors nuclei and the entropy difference represents the entropy carried by the neutron hole coupled to the  $^{106}\text{Cd}$  ( $^{112}\text{Cd}$ ) core.

Finally, the calculated entropy excess in even-odd nuclei compared to near even-even nuclei, according to Eq. (9), is interpreted as the single neutron hole entropy in even-odd nuclei and are plotted in Fig. 3. More information on the calculation procedure can be found in our previous publications [1–3, 11, 13]. An examination of Fig. 3 reveals that the single neutron hole entropy in  $^{105}\text{Cd}$  differs from that in  $^{111}\text{Cd}$ , due to different accessible microstates in each sys-

tem. The different predictions of the number of accessible microstates in two models lead to different results for the single neutron hole entropies. Also, when the number of accessible microstates versus excitation energy oscillates, so does the single neutron hole entropy. The calculated single neutron hole entropy from CT model agrees well with the extracted results in the microcanonical ensemble framework.

In summary, the nuclear level densities for  $^{105,106,111,112}\text{Cd}$  have been extracted within two phenomenological models, the Constant Temperature (CT) and the Back Shifted Fermi Gas (BSFG) models, using the new experimental data on nuclear level density measured by the Oslo group. Then the entropies have been extracted for these nuclei in two models and also in the microcanonical ensemble framework. Finally the entropies of the neutron hole are estimated. The results reveal that the CT model can reproduce microcanonical results for these nuclei better than the BSFG model.

- 
- [1] Razavi R. Role of neutrons and protons in entropy, spin cut off parameters, and moments of inertia. *Phys Rev C*, 2013, **88**: 014316. DOI: [10.1103/PhysRevC.88.014316](https://doi.org/10.1103/PhysRevC.88.014316)
- [2] Razavi R, Behkami A N and Dehghani V. Pairing phase transition and thermodynamical quantities in Sm-148, Sm-149. *Nucl Phys A*, 2014, **930**: 57–62. DOI: [10.1016/j.nuclphysa.2014.07.016](https://doi.org/10.1016/j.nuclphysa.2014.07.016)
- [3] Razavi R, Behkami A N, Mohammadi S, *et al.* Ratio of neutron and proton entropy excess in  $^{121}\text{Sn}$  compared to  $^{122}\text{Sn}$ . *Phys Rev C*, 2012, **86**: 047303. DOI: [10.1103/PhysRevC.86.047303](https://doi.org/10.1103/PhysRevC.86.047303)
- [4] Bethe H A. An attempt to calculate the number of energy levels of a heavy nucleus. *Phys Rev*, 1936, **50**: 332–341. DOI: [10.1103/PhysRev.50.332](https://doi.org/10.1103/PhysRev.50.332)
- [5] Gilbert A and Cameron A G W. A composite nuclear-level density formula with shell corrections. *Can J Phys*, 1965, **43**: 1446–1496. DOI: [10.1139/p65-139](https://doi.org/10.1139/p65-139)
- [6] Zheng H, Giuliani G and Bonasera A. Density and temperature of fermions and bosons from quantum fluctuations. *Nucl Sci Tech*, 2013, **24**: 050512. DOI: [10.13538/j.1001-8042/nst.2013.05.012](https://doi.org/10.13538/j.1001-8042/nst.2013.05.012)
- [7] Guo C C, Sun J and Zhang F S. Comparison between nuclear thermometers in central Xe+Sn collision. *Nucl Sci Tech*, 2013, **24**: 050513. DOI: [10.13538/j.1001-8042/nst.2013.05.013](https://doi.org/10.13538/j.1001-8042/nst.2013.05.013)
- [8] Ma Y G. Application of information theory in nuclear liquid gas phase transition. *Phys Rev Lett*, 1999, **83**: 3617. DOI: [10.1103/PhysRevLett.83.3617](https://doi.org/10.1103/PhysRevLett.83.3617)
- [9] Larsen A C, Ruud I E, Bürger A, *et al.* Transitional  $\gamma$  strength in Cd isotopes. *Phys Rev C*, 2013, **87**: 014319. DOI: [10.1103/PhysRevC.87.014319](https://doi.org/10.1103/PhysRevC.87.014319)
- [10] Ericson T. A statistical analysis of excited nuclear states. *Nucl Phys*, 1959, **11**: 481–491. DOI: [10.1016/0029-5582\(59\)90291-3](https://doi.org/10.1016/0029-5582(59)90291-3)
- [11] Razavi R and Kakavand T. Level studies of  $^{93}\text{Mo}$  via  $^{93}\text{Nb}(p, n\gamma)^{93}\text{Mo}$  reaction and density of discrete levels in  $^{93}\text{Mo}$ . *Nucl Technol Radiat*, 2011, **26**: 69–73. DOI: [10.2298/NTRP1101069R](https://doi.org/10.2298/NTRP1101069R)
- [12] Dilg W, Schantl W, Vonach H, *et al.* Level density parameters for the back-shifted fermi gas model in the mass range  $40 < A < 250$ . *Nucl Phys A*, 1973, **217**: 269–298. DOI: [10.1016/0375-9474\(73\)90196-6](https://doi.org/10.1016/0375-9474(73)90196-6)
- [13] Razavi R, Behkami A N and Mohammadi S. Microscopic study of nuclear level densities and thermodynamical properties in  $^{170,171,172}\text{Yb}$ . *Phys Scripta*, 2012, **86**: 045201. DOI: [10.1088/0031-8949/86/04/045201](https://doi.org/10.1088/0031-8949/86/04/045201)